



# Chapter 17

## The atom

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The energies of the various levels of the hydrogen atom are given by the relation:  $E_n = -\frac{E_0}{n^2}$ , where  $E_0$  is a positive constant and  $n$  is a positive whole number. Given:

$h = 6.6 \times 10^{-34} \text{ J.s}$ ;  $1\text{eV} = 1.6 \times 10^{-19} \text{ J}$ ;  $c = 3 \times 10^8 \text{ m/s}$

A convenient apparatus (D) is used to detect the electrons.

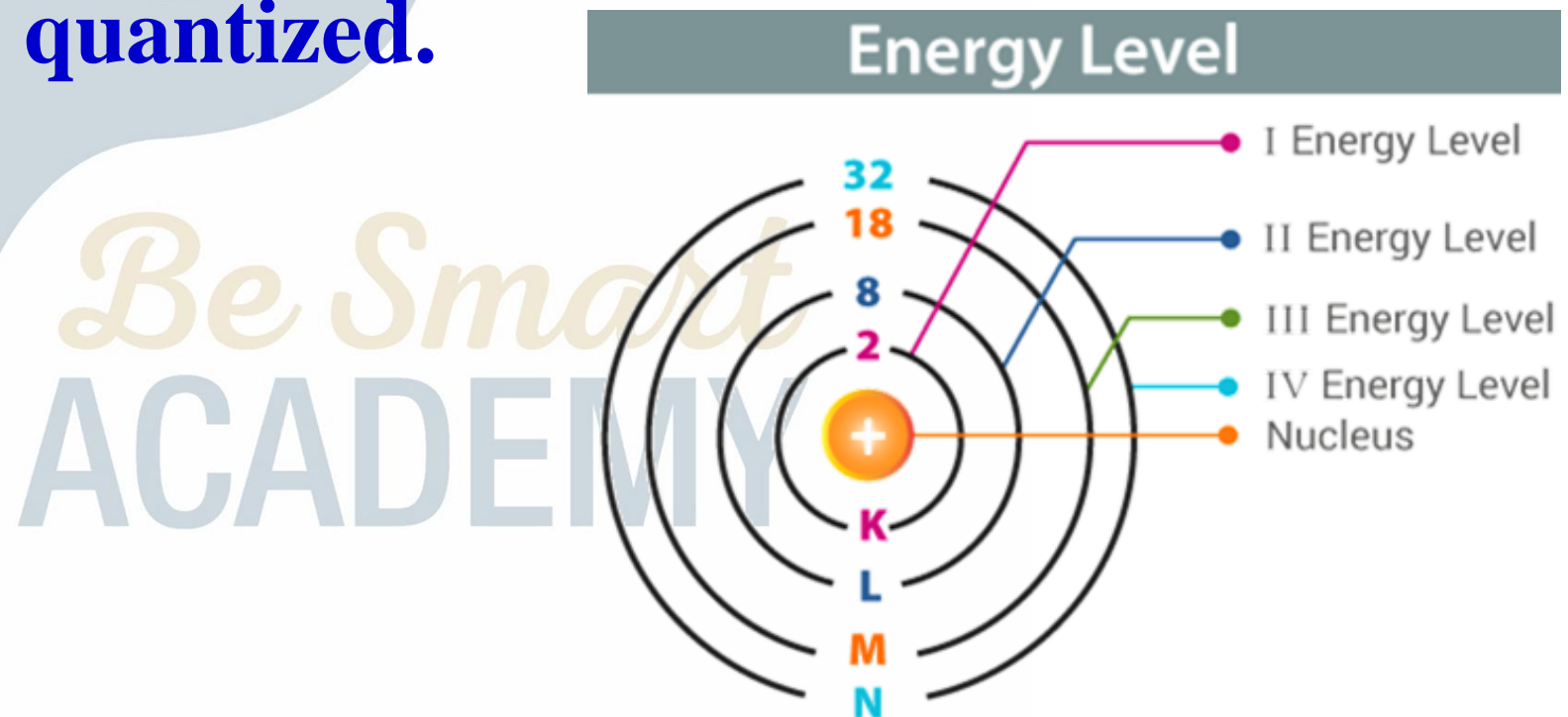
- 1) The energy of the hydrogen atom is quantized. What is meant by “quantized energy”?
- 2) Explain why the absorption or emission spectrum of hydrogen consists of lines.

- 3) A hydrogen atom, initially excited, undergoes a downward transition from the energy level  $E_2$  to the energy level  $E_1$ . It then emits the radiation of wavelength in vacuum:  $\lambda_{2 \rightarrow 1} = 1.216 \times 10^{-7} m$ .**
- a) Determine, in J, the value of the constant  $E_0$ .**
- b) Determine in J the value of the ionization energy of the hydrogen atom taken in its ground state.**

1) What is meant by “quantized energy”?

According to the relation:  $E_n = -\frac{E_0}{n^2}$

The energies of the hydrogen atom can take only well-defined values (discrete), so it is quantized.



2) Explain why the absorption or emission spectrum of hydrogen consists of lines.

For an electronic transition, the emitted photon (or absorbed) has a wavelength

$$\lambda = \frac{hc}{E_m - E_n}$$

Since the energies are quantized, this means that the  $\lambda$  has a well determined value, which corresponds to a line.

Emission Lines



Absorption Lines



$$h = 6.6 \times 10^{-34} \text{ J.s}; 1\text{eV} = 1.6 \times 10^{-19} \text{ J}; c = 3 \times 10^8 \text{ m/s}$$

3) A hydrogen atom, initially excited, undergoes a downward transition from the energy level  $E_2$  to the energy level  $E_1$ . It then emits the radiation of wavelength in vacuum:  $\lambda_{2 \rightarrow 1} = 1.216 \times 10^{-7} \text{ m}$ .

a) Determine, in J, the value of the constant  $E_0$ .

$$E_2 = \frac{E_0}{n^2} = -\frac{E_0}{2^2} = -\frac{E_0}{4} \text{ and } E_1 = \frac{E_0}{n^2} = -\frac{E_0}{1^2} = -E_0$$

$$E_2 - E_1 = -\frac{E_0}{4} - (-E_0) \Rightarrow E_2 - E_1 = \frac{3E_0}{4} = \frac{hc}{\lambda_{2-1}}$$



# Quiz

## Energy levels

Duration: 20min

$$h = 6.6 \times 10^{-34} \text{ J.s}; 1\text{eV} = 1.6 \times 10^{-19} \text{ J}; c = 3 \times 10^8 \text{ m/s}$$

$$\frac{3E_0}{4} = \frac{hc}{\lambda_{2-1}}$$

$$E_0 = \frac{4hc}{3\lambda_{2-1}}$$

$$E_0 = \frac{4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 1.216 \times 10^{-7}}$$

$$E_0 = 2.17 \times 10^{-18} \text{ J}$$



# Quiz

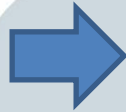
## Energy levels

Duration: 20min

$$h = 6.6 \times 10^{-34} \text{ J.s}; 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}; c = 3 \times 10^8 \text{ m/s}$$

b) Determine in J the value of the ionization energy of the hydrogen atom taken in its ground state

$$E_{ion} = E_{\infty} - E_1$$



$$E_{ion} = 0 - (-E_0)$$

$$E_{ion} = E_0 = 2.17 \times 10^{-18} \text{ J}$$

**4) Among the series of hydrogen is Balmer, which is characterized by the downward transitions from the energy level  $E_p > E_2$  ( $p > 2$ ) to the energy level  $E_2$  ( $n = 2$ ). To each transition  $P \rightarrow 2$  corresponds a line of wave  $\lambda_{p \rightarrow 2}$ .**

**a) Show that  $\lambda_{p \rightarrow 2}$ , expressed in nm is given by  $\frac{1}{\lambda_{p \rightarrow 2}} =$**

$$1.096 \times 10^{-2} \left[ \frac{1}{4} - \frac{1}{p^2} \right].$$

**b) Show that the wavelengths of the corresponding radiations tend, when  $p \rightarrow \infty$ , towards a limit  $\lambda_0$  whose value is to be calculated.**

# Quiz

## Energy levels

Duration: 20min

a) Show that  $\lambda_{p \rightarrow 2}$ , expressed in nm is given by  $\frac{1}{\lambda_{p \rightarrow 2}} =$

$$1.096 \times 10^{-2} \left[ \frac{1}{4} - \frac{1}{p^2} \right].$$

$$E_{ph} = E_p - E_2$$

$$\frac{hc}{\lambda_{p \rightarrow 2}} = -\frac{E_0}{p^2} + \frac{E_0}{2^2}$$

$$\frac{hc}{\lambda_{p \rightarrow 2}} = E_0 \left[ \frac{1}{4} - \frac{1}{p^2} \right]$$

$$\frac{1}{\lambda_{p \rightarrow 2}} = \frac{E_0}{hc} \left[ \frac{1}{4} - \frac{1}{p^2} \right]$$

$$\frac{1}{\lambda_{p \rightarrow 2}} = \frac{2.17 \times 10^{-18}}{6.62 \times 10^{-34} \times 3 \times 10^8} \left[ \frac{1}{4} - \frac{1}{p^2} \right]$$

$$\frac{1}{\lambda_{p \rightarrow 2}} = 1.096 \times 10^{-2} \left[ \frac{1}{4} - \frac{1}{p^2} \right]$$

b) Show that the wavelengths of the corresponding radiations tend, when  $p \rightarrow \infty$ , towards a limit  $\lambda_0$  whose value is to be calculated

$$\frac{1}{\lambda_{p \rightarrow 2}} = 1.096 \times 10^{-2} \left[ \frac{1}{4} - \frac{1}{p^2} \right]$$

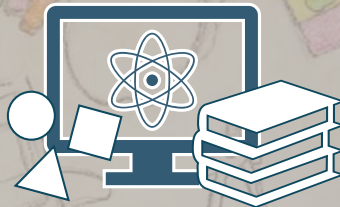
$$\frac{1}{\lambda_0} = 1.096 \times 10^{-2} \left[ \frac{1}{4} - \frac{1}{\infty^2} \right] \Rightarrow \frac{1}{\lambda_0} = 1.096 \times 10^{-2} \left[ \frac{1}{4} - 0 \right]$$

$$\frac{1}{\lambda_0} = 0.274 \times 10^{-2}$$

$$\lambda_0 = 364.9 \text{ nm}$$



# The End







$$h = 6.6 \times 10^{-34} \text{ J.s}; 1\text{eV} = 1.6 \times 10^{-19} \text{ J}; c = 3 \times 10^8 \text{ m/s}$$

The energy levels of the hydrogen atom are given by  $E_n = -\frac{13.6}{n^2}$ , where  $n$  is whole non-zero number and  $E_n$  in eV.

1. The energy of the energy levels of the hydrogen atom are quantized. Justify.
2. Calculate the energy of the hydrogen atom in its fundamental state.
3. Show that when the hydrogen atom passes from an energy level  $E_q$  to another level  $E_p$  less than  $q$ , it releases energy under a certain form to be specified.

$$h = 6.6 \times 10^{-34} \text{ J.s}; 1\text{eV} = 1.6 \times 10^{-19} \text{ J}; c = 3 \times 10^8 \text{ m/s}$$

1. The energy of the energy levels of the hydrogen atom are quantized. Justify.

The energy of the energy levels depends on the number  $n$  (whole number), then only a set of well-defined values is allowed, then they are quantized.

2. Calculate the energy of the hydrogen atom in its fundamental state.

In the fundamental state  $n = 1$ :

$$E_1 = -\frac{13.6}{n^2} = -\frac{13.6}{(1)^2}$$
$$E_1 = -13.6 \text{ eV}$$

3. Show that when the hydrogen atom passes from an energy level  $E_q$  to another level  $E_p$  less than  $q$ , it releases energy under a certain form to be specified.

When the hydrogen atom passes from higher energy level to lower energy level; its energy decreases.

This energy appears as radiant energy carried by the emitted photon.

4. We intend to study the set of radiations emitted through the downwards transition towards  $p = 2$ .

a) Show that the wavelengths, in  $\mu m$ , of the radiations emitted by the hydrogen atom during these transitions are given by:  $\lambda = \frac{0.365}{1 - \frac{4}{q^2}}$  where  $q$  is a whole number ( $q \geq 3$ ).

$$E_{ph} = E_h - E_l \quad \Rightarrow \quad \frac{hc}{\lambda} = -\frac{13.6}{q^2} - \left( -\frac{13.6}{2^2} \right)$$

$$\frac{hc}{\lambda} = \frac{13.6}{4} \left[ 1 - \frac{4}{q^2} \right]$$

$$\frac{hc}{\lambda} = \frac{13.6}{4} \left[ 1 - \frac{4}{q^2} \right]$$

$$\frac{hc}{\lambda} = 3.4 \left[ 1 - \frac{4}{q^2} \right]$$

$$\lambda = \frac{hc}{3.4 \left[ 1 - \frac{4}{q^2} \right]}$$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3.4 \times 1.6 \times 10^{-19} \left[ 1 - \frac{4}{q^2} \right]}$$

$$\lambda = \frac{19.86 \times 10^{-26}}{5.44 \times 10^{-19} \left[ 1 - \frac{4}{q^2} \right]}$$

$$\lambda = \frac{3.65 \times 10^{-7}}{\left[ 1 - \frac{4}{q^2} \right]}$$

$$\lambda(\mu m) = \frac{3.65 \times 10^{-7}}{\left[ 1 - \frac{4}{q^2} \right]} \times 10^6 = \frac{0.365}{\left[ 1 - \frac{4}{q^2} \right]}$$

b) Knowing that the wavelengths  $\lambda$  of the visible spectrum belongs to  $[0.4\mu m; 0.75\mu m]$ . Show that the emission spectrum of the hydrogen atom holds four visible rays for 4 different values of  $q$  whose values are to be determined.

$$\lambda_{vi} \leq \lambda \leq \lambda_{red} \quad \Rightarrow \quad 0.4\mu m \leq \frac{0.365}{\left[1 - \frac{4}{q^2}\right]} \leq 0.75\mu m$$

$$\frac{0.4\mu m}{0.365} \leq \frac{1}{\left[1 - \frac{4}{q^2}\right]} \leq \frac{0.75\mu m}{0.365}$$



$$1.095 \leq \frac{1}{\left[1 - \frac{4}{q^2}\right]} \leq 2.054$$

$$\frac{1}{1.095} \geq 1 - \frac{4}{q^2} \geq \frac{1}{2.054}$$

$$\frac{1}{1.095} - 1 \geq 1 - \frac{4}{q^2} - 1 \geq \frac{1}{2.054} - 1$$

$$-0.086 \geq -\frac{4}{q^2} \geq -0.513$$

$$-0.086 \geq -\frac{4}{q^2} \geq -0.513$$

$$\left(-\frac{1}{4}\right) - 0.086 \geq \left(-\frac{1}{4}\right) - \frac{4}{q^2} \geq \left(-\frac{1}{4}\right) - 0.513$$

$$0.0215 \leq \frac{1}{q^2} \leq 0.1282$$

$$\frac{1}{0.0215} \geq q^2 \geq \frac{1}{0.1282}$$

$$\frac{1}{0.0215} \geq q^2 \geq \frac{1}{0.1282}$$

$$46.5 \geq q^2 \geq 7.80$$

$$6.77 \geq q \geq 2.79$$

$q$  is whole number then:  $q \in \{3, 4, 5, 6\}$   
This corresponds to 4 transitions

c) Determine the wavelengths of the visible radiations in the Balmer's series.

$$\lambda(\mu m) = \frac{0.365}{\left[1 - \frac{4}{q^2}\right]}$$

For  $q = 3$ :

$$\lambda(\mu m) = \frac{0.365}{\left[1 - \frac{4}{(3)^2}\right]} = \frac{0.365}{\left[1 - \frac{4}{9}\right]}$$

$$\lambda(\mu m) = 0.657 \mu m$$

For  $q = 4$ :

$$\lambda(\mu m) = \frac{0.365}{\left[1 - \frac{4}{(4)^2}\right]} = \frac{0.365}{\left[1 - \frac{4}{16}\right]}$$

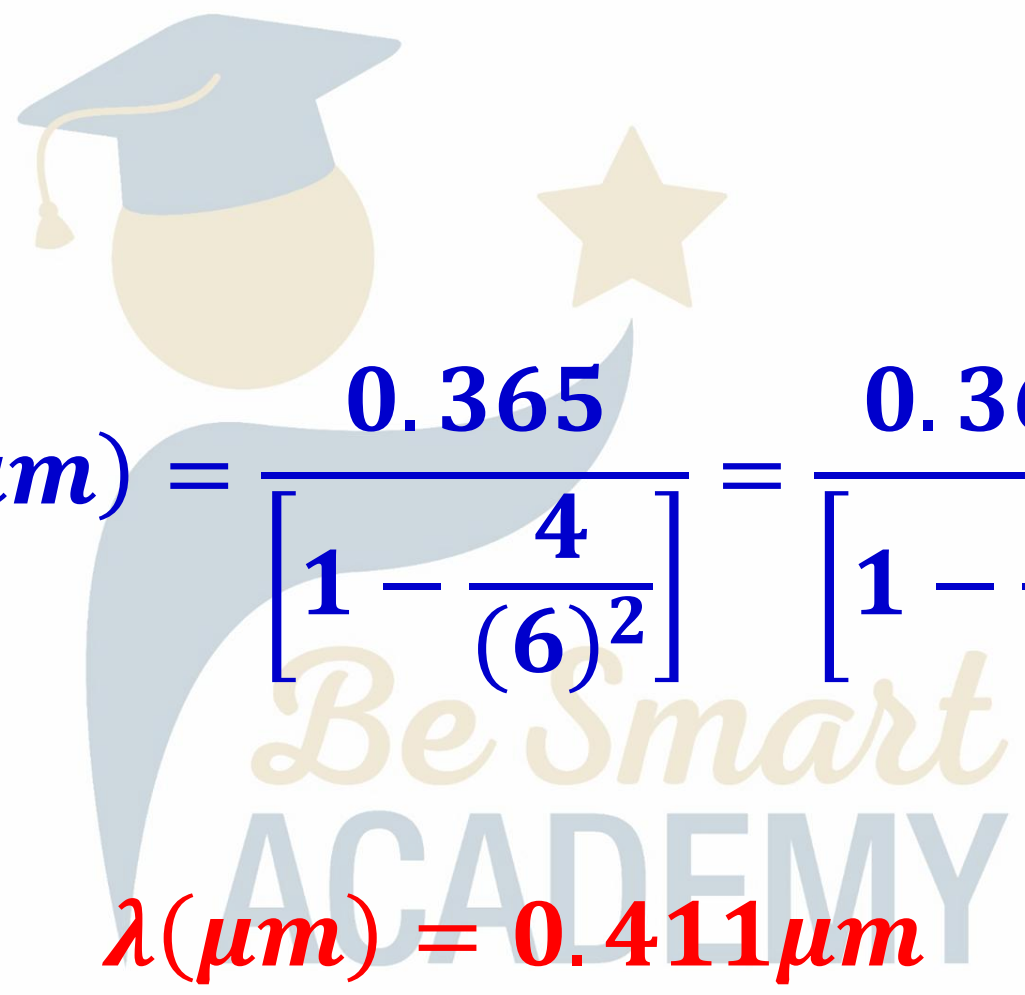
$$\lambda(\mu m) = 0.487 \mu m$$

For  $q = 5$ :

$$\lambda(\mu m) = \frac{0.365}{\left[1 - \frac{4}{(5)^2}\right]} = \frac{0.365}{\left[1 - \frac{4}{25}\right]}$$

$$\lambda(\mu m) = 0.435 \mu m$$

For  $q = 6$ :


$$\lambda(\mu m) = \frac{0.365}{\left[1 - \frac{4}{(6)^2}\right]} = \frac{0.365}{\left[1 - \frac{4}{36}\right]}$$

$$\lambda(\mu m) = 0.411 \mu m$$

5. The hydrogen atom being in the 1<sup>st</sup> excited state receives an electron carrying a kinetic energy of 2.9eV.

a) Specify the level that the atom cannot overpass.

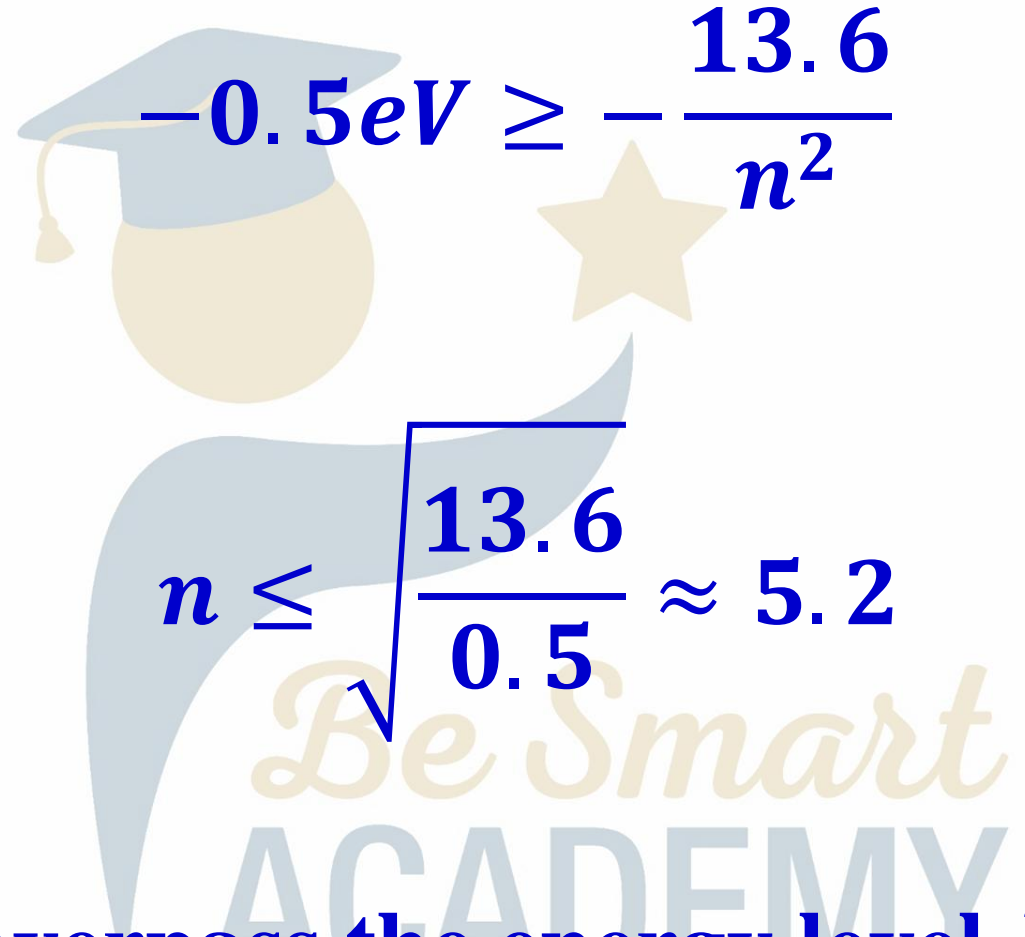
The energy of the atom in the 1<sup>st</sup> excited state is:

$$E_2 = -\frac{13.6}{2^2} = -\frac{13.6}{4} = -3.4eV$$

$$KE_e + E_2 = 2.9 + (-3.4eV) = -0.5eV$$

$$-0.5eV \geq -\frac{13.6}{n^2}$$




$$-0.5\text{eV} \geq -\frac{13.6}{n^2}$$

$$n \leq \sqrt{\frac{13.6}{0.5}} \approx 5.2$$

The atom will not overpass the energy level  $E_5$

b) Indicate the transition that corresponds to the electron carrying the minimum kinetic energy after interaction and then calculate its value.

The electron carries minimum kinetic energy if the atom absorbs the maximum possible energy.

It corresponds to the transition between  $E_2$  towards  $E_5$

$$\Delta K.E = E_5 - E_2 = -\frac{13.6}{5^2} - \left(-\frac{13.6}{2^2}\right)$$

$$\Delta K.E = -0.544 + 3.4$$

$$\Delta K.E = 2.86 eV$$

The electron carries after interaction:

$$KE_{after} = KE - \Delta KE$$

$$KE_{after} = 2.9 - 2.86$$

$$KE = 0.04 eV$$

6. The number of hydrogen atom  $N_n$  in the energy level  $E_n$  is given by  $N = N_1 e^{-\frac{(E_n - E_1)}{kT}}$  where T is the absolute temperature in Kelvin K, k is Boltzmann's constant  $k = 1.38 \times 10^{-23} SI$  &  $N_1$  is the population of atoms in the ground state.

a) Complete the following table.

	T = 3000K (red or cold stars)	T = 8000K (white stars)
$\frac{N_2}{N_1}$		

$$N = N_1 e^{\frac{-(E_n - E_1)}{kT}} \quad \Rightarrow \quad \frac{N}{N_1} = e^{\frac{-(E_n - E_1)}{kT}}$$

For  $T = 3000K$

$$\frac{N}{N_1} = e^{\frac{-(-3.4 - (-13.6)) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 3000}}$$

$$\frac{N}{N_1} = e^{\frac{-16.32 \times 10^{-19}}{4140 \times 10^{-23}}} = e^{-39.42} = 7.6 \times 10^{-18}$$

**For  $T = 8000K$**

$$\frac{N}{N_1} = e^{\frac{-(-3.4 - (-13.6)) \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 8000}}$$

$$\frac{N}{N_1} = e^{\frac{-16.32 \times 10^{-19}}{11040 \times 10^{-23}}} = e^{-14.78} = 3.8 \times 10^{-7}$$

	<b>T = 3000K</b> (red or cold stars)	<b>T = 8000K</b> (white stars)
$\frac{N_2}{N_1}$	<b><math>7.6 \times 10^{-18}</math></b>	<b><math>3.8 \times 10^{-7}</math></b>

b) Specify the star whose spectrum is rich in the radiation that falls in the Balmer's series.

$$\frac{N_2}{N_1} (\text{white star}) \gg \frac{N_2}{N_1} (\text{red star})$$

In white stars there is a big number of atoms in the first excited state, so they are expected to be rich in radiations that fall in Balmer's series



A top-down view of several children sitting around a large table, drawing on a massive sheet of paper. The paper is covered with various hand-drawn illustrations related to school and learning. In the center, a large schoolhouse with a clock tower is being drawn. To the left, there's a drawing of a microscope, a lightbulb, and a person sitting at a desk. To the right, a hopscotch grid with numbers 1 through 9 is visible, along with a drawing of a person and a stack of books. At the bottom, a calculator and a magnifying glass over the number 3 are drawn. The children are using various colored pencils and markers to complete their drawings.

# The End

